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Beyond the Metre (Part III)

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BEYOND THE METRE (Part III)

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Introduction

This article is the third in a series discussing the wide spectrum of metric units. The first dealt with some general aspects of the International System of Units (SI) and then went on to consider the metre, square metre, cubic metre and second. The second article described the kilogram and the metric units of velocity and acceleration. This article deals with metric units of density, force and pressure.

The Kilogram Per Cubic Metre (kg/m³)

The concept of density originated in the Middle Ages. (1) By itself, the term refers to the mass contained in a single volume unit of the material under consideration. We calculate density by measuring the mass of a chunk of the substance and dividing the result by the volume it occupies. This may be expressed

$$\text{density} = \frac{\text{mass of sample of substance}}{\text{volume of sample of substance}}$$

One cubic metre of water, for example, has a mass of one tonne or 100 kg. Using SI units, its density would be

$$\text{density of water} = \frac{1000\ \text{kg}}{1\ \text{m}^3}$$

As shown in the previous article of this series, the expression above may be broken into numerical and label parts. Performing the arithmetic indicated yields

$$\text{density of water} = 1000\ \frac{\text{kg}}{\text{m}^3} = 1000\ \text{kg/m}^3$$

Recalling the coherence of SI units, we can see that the SI unit of density is the *kilogram per cubic metre.*

The density of gold is more than 19,000 kg/m³. Gases at atmospheric pressure and common temperatures typically have densities of a few kilograms per cubic metre, somewhat larger than one might guess.

Density may also be described by comparing the mass of the material under discussion to the mass of an equal volume of some standard substance, such as water. This sort of ratio is called *specific gravity*, a term which may be familiar to persons who test automobile batteries or brew their own beer. The specific gravity of gold, for example, is

$$
\begin{aligned}
\text{specific gravity of gold} &= \frac{\text{mass of 1 m}^3\text{ of gold}}{\text{mass of 1 m}^3\text{ of water}} \\
&= \frac{19{,}000\ \text{kg/m}^3}{1{,}000\ \text{kg/m}^3} \\
&= 19\,\frac{\text{kg/m}^3}{\text{kg/m}^3} \\
\text{specific gravity of gold} &= 19
\end{aligned}
$$

When SI labels appear in both sides of a fraction, they may be removed. In this case, the operation eliminates all the units present. Specific gravity is a pure number with no associated units. Its value for a particular substance is the same in *any* measurement system, which can be a handy advantage. Furthermore, the specific gravities of common materials with reference to water are usually convenient numbers around unity.

Measurement systems may be designed so that the specific gravity equals the numerical value of the density of any substance in the system. Expressing metric densities in terms of litres instead of cubic metres results in this convenience. In SI, however, the litre is not to be used in compounding units, and so specific gravity values differ from densities.

The widest range of densities occurs in astronomy. A gas of density 10^{-18} kg/m³ fills the space between the stars. Neutron star densities, on the other hand, are roughly 10^{18} kg/m³.

Many college students have no adequate conception of density. Many merely memorize the incantation "mass per unit volume." Some of the difficulty even at this late age is in cognitive development, muddled further by a language which describes a pound of gold as "heavier" than a pound of lead. The 1979 *World Almanac* contains a table entitled "Density of Gases (kilograms per cubic metre)" which lists the values under a column headed "Wgt."

The Newton (N)

Friction rubs up against our lives in so many ways that everyday objects would seem to behave strangely without it. Via television, space exploration has pulled the frictionless environment of orbit into the living room. Let us imagine ourselves in space with a 2 lb can of coffee, far from the sun or any other powerful gravitational source. How will the can behave?

Place the can at rest and watch it. It sits without moving. If there are no stars or planets or anything else in the vicinity, how can the observer be sure the can is really at rest? The easiest solution is to say that the can is not moving with respect to the observer. Jet plane passengers do this whenever they calmly pour themselves a drink of soda pop while cruising at 1000 km/h. This journal may seem to be at rest to you, but it's actually zipping around the sun at nearly 30 km/s.

The observer gives the can of coffee a brief push. It moves away. There is no friction in space to slow it down. Unless the observer catches it, the can will keep on moving slowly away forever. Nothing is making the can pick up speed either, once the observer stops pushing it. It moves away at constant velocity.

The Renaissance physicist Galileo first recognized this situation, but today's student usually learns these results in the form Isaac Newton recounted them:

> **A body at rest will remain at rest, and a body in motion in the same state of motion unless acted upon by an outside force.**

Imagine a can sailing by our space observer. Suddenly, it slows down. It has not remained in the same state of motion. According to Newton, a force of some kind must be acting on the can. In this sense, Newton's statement, which is called his "first law," *defines* force. We know a force is present when an object changes its velocity.

A change in velocity is an acceleration, which SI measures in metres per second per second. Is there any way to predict how big a velocity change will occur when our observer pushes the coffee can?

Newton answered this question in his second law, expressed in modern terms as "A given force accelerates an object in inverse proportion to its mass." This may be written symbolically as

$$\textbf{force} = \textbf{(mass of object)(acceleration produced by force)}$$

or simply

$$\mathbf{F} = \mathbf{m\,a}$$

All a physics student needs to do is plug in the values and then grind out the calculation.

For a given object under normal circumstances (such as the coffee can) the mass is constant. If we double the force on it by pushing it twice as hard, the can will pick up speed twice as fast while being pushed. Applying the same force to objects of different mass accelerates them differently.

Suppose a 2 lb object is accelerated 2 ft. per second per second. The formula gives

$$\textbf{force} = \textbf{(2 lbs.)(2 ft. per second per second)}$$

Arithmetic reveals that the force is "4", but four *what*? Suppose the mass were 2 g and the acceleration 2 miles per hour each minute. The answer is "4" again, but it is not likely to be the same "4". In order for the result to mean anything, its units must be clear.

One big advantage of SI is that it is coherent. Its units work together when used in a formula. If we measure the mass and acceleration in SI units, the force will be calculated in SI units as well.

Our coffee can has a mass of about a kilogram. If we push it just hard enough to get it traveling one metre per second faster each second, after one second it has a speed of 1 m/s. After another second it is moving 2 m/s. At the end of the third it is moving 3 m/s. If we plug this acceleration and the kilogram mass into the formula, we have

$$\mathbf{F} = \mathbf{m\,a}$$

$$\text{force on can} = (\text{mass of can})(\text{rate can gains speed})$$

$$= (1\ \text{kg})(1\ \text{m/s}^2)$$

The numbers are easy enough to multiply, and we have shown that the labels can be treated just like numbers, in this case analagous to multiplying a fraction (m/s^2) times a whole number (kg). So we obtain

$$\text{force on can} = 1\ \text{kg}\ \ \text{m/s}^2$$

The SI unit of force is presumably the *kilogram-metre per second squared.* This string of syllables is inconvenient to pronounce and does not accept metric prefixes elegantly. The General Conference of Weights and Measures has therefore given the unit a name all its own. (2) In honor of the man who first worked out the relation between force and acceleration and mass, the unit is called the *newton.*

Units which, as the newton has done, emerge from applying formulae are called *derived* units. (2) They may not have special names. The cubic metre and the kilogram per cubic metre are derived units.

The name of the newton is written out in lower case to distinguish it from Newton's name, but its symbol is an upper case N. All but one of the SI units named in honor of individuals follow the same pattern. So many SI units are named for British scientists that the casual observer might expect SI to be an English system.

Spring scales respond to the force that pulls their springs. They should be calibrated in newtons, not in kilograms.

Since Galileo's time it has been known that all objects fall due to the pull of gravity on them with the same acceleration near the earth's surface. By Newton's law, the force of gravity which produces this motion must be given by the product of the object's mass and this gravitational acceleration, which is about 9.8 m/s^2. The force of gravity on a kilogram of matter is then given by

$$\mathbf{F} = \mathbf{m\,a} = (1\ \text{kg})(9.8\ \text{m/s}^2) = (9.8\ \text{kg}\ \ \text{m/s}^2) = 9.8\ \text{N}$$

The pull of gravity on a 1 kg mass is about 10 newtons.

If you hold a 100 g object such as a flashlight battery, you are exerting a force of 1 newton on that object. The force of gravity on a falling apple is about a newton, appropriately enough.

One "metric" unit that should *never* be used is the "kilogram-force", based on the pull of gravity upon a one-kilogram mass. The kilogram-force makes the kilogram seem like a force unit. It is not coherent with other SI units. It varies from place to place like the pull of gravity. Trying to puzzle out the meaning of the term has been known to confuse students and college professors alike. The only solution is to never bring it up. Throw away spring scales calibrated in "kilograms." Using such misleading instruments in a classroom is like maintaining a dictionary in which the definitions have been mismatched to the words.

The Pascal (Pa)

The concept of pressure is related closely to force, and it turns up often in everyday life. Neglecting to check automobile tire pressures can result in needless wear and low gasoline mileage. Neglecting to check one's blood pressure can result in needless wear and low longevity. Television weather reporters carefully inform viewers of the barometric pressure.

It is not clear from everyday experience exactly what pressure is. Even commonly encountered units of pressure seem to be measuring different quantities. Tire pressures are monitored in "pounds per square inch," blood pressures in "millimetres of mercury," and barometric pressures in "inches of mercury" or perhaps "millibars." European tire gauges are labeled in "kilograms per square centimetre" and other units of pressure include the torr, the atmosphere and the inch of water.

Imagine spreading 20 layers of aluminum foil on a tabletop one metre square. The foil will exert some pressure at each point on the table. Now fold the layers of foil in half. This doubles the foil's pressure on one half of the table (while the pressure due to the foil drops to zero on the other half).

The pull of gravity on the foil is exerting a force on the table. Instead of being applied at just one point, the force of gravity is spread across the tabletop under the foil. Folding the foil doesn't change the force of gravity, it is just concentrating it more on one part of the table.

Now add more foil to make the layer twice as thick. This will clearly double the pressure of the foil on the table. There is more foil for gravity to pull and so the force on the table top is also twice as great.

These mental experiments suggest that pressure involves a force and an area to which it is applied. Increasing the force increases the pressure. Increasing the area decreases the pressure proportionately. This situation may be described mathematically as

$$\text{pressure} = \frac{\text{force applied}}{\text{area force applied to}}$$

And this definition is born out by the name of a common customary unit of pressure, the pound per square inch.

What is the metric unit of pressure? The millimetre of mercury may sound metric, but it does not have the form indicated above. Force is measured in newtons in SI, and area in square metres. The appropriate SI unit of pressure would then be the *newton per square metre* (N/m^2).

This is a small unit. Imagine dropping the one-newton apple mentioned in the last section into a food processor and spreading the applesauce which results evenly across the tabletop. The pressure at each point under the sauce would be one newton per square metre.

Writing out "newton per square metre" becomes so tedious that the GCPM has given the unit its own name, the *pascal;* in honor of the tormented French genius Blaise Pascal (both scientist *and* literary giant) who invented the barometer. The pascal's symbol Pa consists of two letters. Although less elegant than a single letter, it combines in the same way with the prefix symbols. It must be capitalized.

Instead of spreading out the applesauce, we might keep it in a smaller area and increase the pressure beneath it. Contained in a light aluminum can with an end of area 10 cm^2, the pressure underneath the sauce would now be

$$\textbf{pressure of sauce} = \frac{\textbf{1 newton}}{\textbf{10 square centimetres}} = \frac{\textbf{1 N}}{\textbf{0.001 m}^2} = \textbf{1000 Pa}$$

The force on the applesauce has not changed at all. This major increase in pressure has come solely from confining the force into a smaller area.

As pressures go, a kilopascal still is not very great. A toy balloon may hold air at a pressure 10 kPa higher than its surroundings. An automobile tire regularly needs 200 kPa of pressure. Scientists studying materials under very high pressure conditions commonly work in the gigapascal range.

We breathe air at a pressure of about 100 kPa. Air pressure is important in accurate weather prediction. We can calculate the total force that air exerts on the side of a house. If the wall measures 10 m by 20 m, its area is 200 m^2. On each square metre, air at 100 kPa exerts 100,000 N of force. The total force on the wall is then 20,000,000 newtons, or 20 meganewtons. Although the newton is small, this is a major force, enough to lift 2,000,000 kg. The house itself does not weigh this much, but it doesn't sail away because the force on this side is opposed by the air pressure pushing on the other side of the wall. Unbalanced air pressure can produce spectacular results. A one-sided pressure drop of 10% or so in tornadoes has been observed to result in the explosion of buildings. Air pressure decreases with altitude. At 80 km it is down to one pascal.

Feeling the pressure yourself? Here is a way to measure just how much pressure your feet are under. Step onto a piece of newsprint and draw around your shoes. Measure the area of your soles with a transparent square centimetre grid or by drawing such squares right on the newspaper. Count the squares, giving a value of ½ for each partial square and 1 for each square fully enclosed by the outlines. Divide by 10,000 to convert this number into square metres. The pull of gravity on you in newtons is about ten times your mass in kilograms. Divide this value by the area of your soles in square metres to find the pressure in pascals. If your feet hurt at the end of the day, now you know why. Calculate the pressure on your ankles to see why they are subject to injury.

The subject of pressure is a current battleground between advocates of SI and proponents of older, non-coherent measurements. (3) Meteorologists wish to keep the *bar*, which is used in weather forecasting. Physicians support retention of the *millimetre of mercury* (4), which refers to the pressure under a column of mercury 1 mm high. Normal human blood pressure in this unit is described with two values: 120 and 80. Physicians find the unit convenient because they measure blood pressure with a device which actually uses a column of mercury. But how do such values relate to other pressures? It is clearer to describe the pressures as about 16 and 10 kPa, roughly the pressure in a balloon. The name "millimetre of mercury" sounds more like a measure of height than pressure and so obscures the nature of the quantity it purports to describe. Despite its long history, it is absolutely useless in combination with any other sorts of measures.

Doctors will probably capitulate sooner or later, but the switch toward SI with this quantity is moving very slowly for now. Tires marked in kilopascals have been declared safety hazards. Americans evidently feel little pressure to speed up metrication.

Summary

This article has reviewed the kilogram per cubic metre, the newton, and the pascal. The next article in this series will discuss SI units of energy, power and temperature.

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Beak of the Week

Research has shown that woodpeckers, at the peak of percussion, strike their beaks against trees at 1,300 mph. Upon impact, the birds' heads snap back with a force of 1,000 g's. Why don't they knock themselves out?